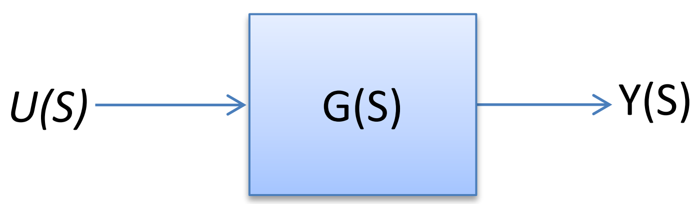
Stability of Control System

References:

1. Modern control Engineering by OGATA
2. Automatic control system by KUO.
3. Control system Engineering by Norman S. Nise.
4. http://imtiazhussainkalwar.weebly.com.

Stability is the most important system specification. If a system is unstable transient response and steady-state errors are moot points. An unstable system cannot be designed for a specific transient response or steady-state error requirement.

Transfer Function: The transfer function G(S) of the plant is ratio of Laplace transform of output to the Laplace transform of input considering initial conditions to zero.



Transfer function helps us to check:

1. The stability of the system
2. Time domain and frequency domain characteristics of the system
3. Response of the system for any given input

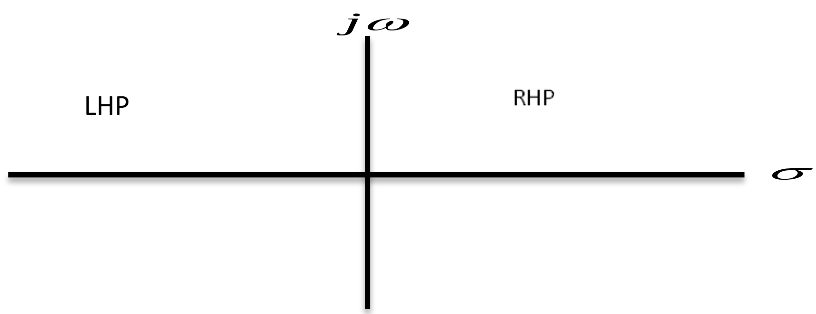
Transfer Function has the form:

$$\frac{Y(s)}{X(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

Then,

- Roots of denominator polynomial of a transfer function are called ‘poles’.

- Roots of numerator polynomial of a transfer function are called ‘zeros’.
- The poles and zeros of the system are plotted in s-plane to check the stability of the system.



s-plane

- If all the poles of the system lie in left half plane the system is said to be Stable.
- If any of the poles lie in right half plane the system is said to be unstable.
- If pole(s) lie on imaginary axis the system is said to be marginally stable.
- The location of a pole in the complex plane is denoted symbolically by a cross (x), and the location of a zero by a small circle *(0).* The s-plane including the locations of the finite poles and zeros of $\boldsymbol{F(s)}$ is called the **pole-zero map** of $\boldsymbol{F(s)}$ **. A** similar comment holds for the z-plane.

Complex-Variable Concept

A complex variable s has two components: a real component σ and an imaginary component ω. Graphically, the real component of s is represented by a σ axis in the horizontal direction, and the imaginary component is measured along the vertical $j\omega$ axis, in the complex s-plane. Figure A-1 illustrates the complex s-plane, in which any arbitrary point $s = s_1$ is defined by the coordinates $\sigma = \sigma_1$, and $\omega = \omega_1$, or simply $s_1 = \sigma_1 + j\omega_1$.

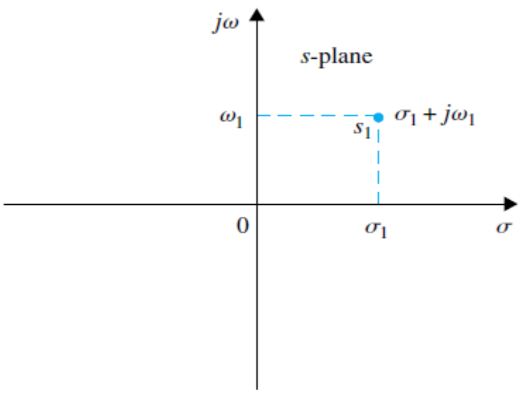


Figure A-1 The complex s-plane.

<u>COMPLEX PLANE POLEZERO MAPS:</u>

The rational functions F (s) for continuous systems can be rewritten as

$$F(s) = \frac{b_m \sum_{i=0}^{m} (b_i/b_m) s^i}{\sum_{i=0}^{n} a_i s^i} = \frac{b_m \prod_{i=1}^{m} (s + z_i)}{\prod_{i=0}^{n} (s + p_i)} = \text{T.F}$$

where the terms s + zi are factors of the numerator polynomial and the terms s $+p_i$, are factors of the denominator polynomial.

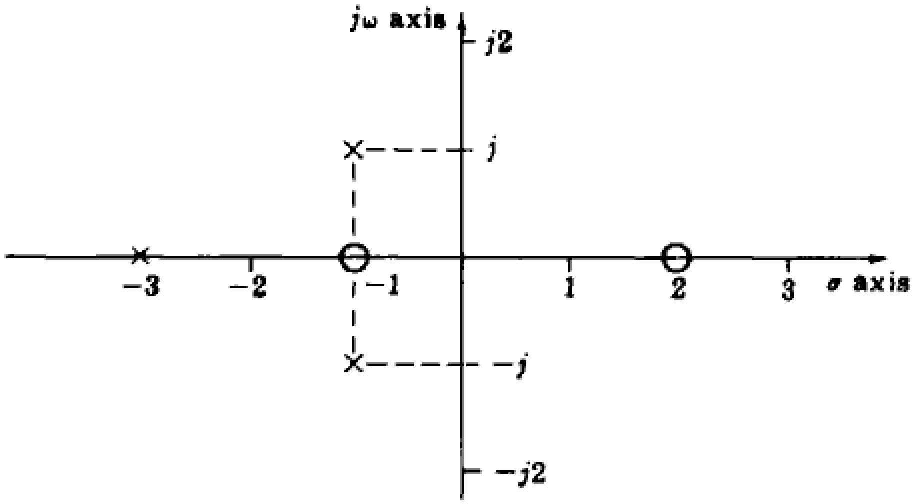
EX 1: Let ***F (s)*** be given by

$$F(s) = \frac{2s^2 - 2s - 4}{s^3 + 5s^2 + 8s + 6}$$

$$F(s) = \frac{2(s+1)(s-2)}{(s+3)(s+1+j)(s+1-j)}$$

which can be rewritten **as:**

F (s) has finite zeros at s = - 1 and s = 2. F(s) has finite poles at s = - 3, s = - 1 - j , and s = - l+j. The pole-zero map of ***F (s)*** is shown in Fig. below



There are two kinds of stability definitions in control system study.

- ❒ Absolute Stability:

refers to the condition of stable or unstable.

- ❒ Relative Stability: Once the system is found to be stable, it is of interest to

determine how stable it is, and the degree of stability is measured by relative **stability.**

<u>Methods of determining stability</u>

1. ***Routh–Hurwitz criterion***[1]: an algebraic method that provides information on the absolute stability of a linear time-invariant system. The criterion tests whether any roots of the characteristic equation lie in the right half of the *s*-plane. The number of roots that lie on the imaginary axis and in the right half of the *s*-plane is also indicated.

2. *Nyquist criterion*[6]: a semigraphical method that gives information on the difference between the number of poles and zeros of the closed loop transfer function by observing the behavior of the Nyquist plot of the loop transfer function. The poles of the closed-loop transfer function are the roots of the characteristic equation. This method requires that we know the relative location of the zeros of the closed-loop transfer function.

3. *Root locus plot* (Chapter 8): represents a diagram of loci of the characteristic equation roots when a certain system parameter varies. When the root loci lie in the right half of the s-plane, the closed-loop system is unstable.

4. *Bode diagram* (Appendix A): the Bode plot of the loop transfer function $G(s)H(s)$ may be used to determine the stability of the closed-loop system. However, the method can be used only if $G(s)$ $H(s)$ has no poles and zeros in the right-half s-plane.

5. *Lyapunov's stability criterion*: a method of determining the stability of nonlinear systems, although it can also be applied to linear systems. The stability of the system is determined by checking on the properties of the *Lyapunov function* of the system.

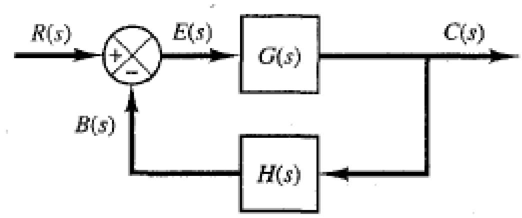
Characteristic Equation:

It is the equation formed by putting the denomenter of the T.F of the system equal to zero.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = closed - loop\ transferfunction = T.F$$

The Characteristic Equation (C.E) of the system is given by:

$$1+G(s)H(s)=0$$



Most linear closed-loop systems have closed-loop transfer functions of the form:

$$\frac{C(s)}{R(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1}s + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n}$$

A simple criterion, known as Routh's stability criterion, enables us to determine the number of closed-loop poles that lie in the right-half s plane without having to factor the denominator polynomial.

Routh's Stability Criterion:

Routh's stability criterion tells us whether or not there are unstable roots in a polynomial equation without actually solving for them. This stability criterion applies to polynomials with only a finite number of terms. When the criterion is applied to a control system, information about absolute stability can be obtained directly from the coefficients of the characteristic equation. The procedure in Routh's stability criterion is as follows:

1. Write the polynomial in s in the following form:

$$a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n = 0$$

where the coefficients are real quantities. We assume that $a_n \neq 0$; that is, any zero root has been removed.

2. If any of the coefficients are zero or negative in the presence of at least one positive coefficient, there is a root or roots that are imaginary or that have positive real parts. Therefore, in such a case, the system is not stable.

3. If all coefficients are positive, arrange the coefficients of the polynomial in rows and columns according to the following pattern:

$$\begin{array}{llllll}
s^n & a_0 & a_2 & a_4 & a_6 & \cdots \\
s^{n-1} & a_1 & a_3 & a_5 & a_7 & \cdots \\
s^{n-2} & b_1 & b_2 & b_3 & b_4 & \cdots \\
s^{n-3} & c_1 & c_2 & c_3 & c_4 & \cdots \\
s^{n-4} & d_1 & d_2 & d_3 & d_4 & \cdots \\
\cdot & \cdot & \cdot & & &
\end{array}$$

The process of forming rows continues until we run out of elements. (The total number of rows is n + 1.) The coefficients *b1, b2, b3,* and so on, are evaluated as follows:

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}$$

$$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$$

$$b_3 = \frac{a_1 a_6 - a_0 a_7}{a_1}$$

Then continue solve for b_4 and b_5 in the same way until we obtain zero. The same pattern of cross- multiplying the coefficients of the two previous rows

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1}$$

$$c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1}$$

$$c_3 = \frac{b_1 a_7 - a_1 b_4}{b_1}$$

The table is continued horizontally and vertically until only zeros are obtained. Any row can be multiplied by a positive constant before the next row is computed without disturbing the properties of the table.

- If all the constants in the first column have the same sign (positive or negative) then, the system is stable.
- If the first column have (positive and negative) then, the system is unstable.
- If one of these values equal to zero then the system is critical and the value of critical amplifier gain can be obtained accordingly.

EX 1: apply Routh's stability criterion to the following third-order polynomial:

$$a_0 s^3 + a_1 s^2 + a_2 s + a_3 = 0$$

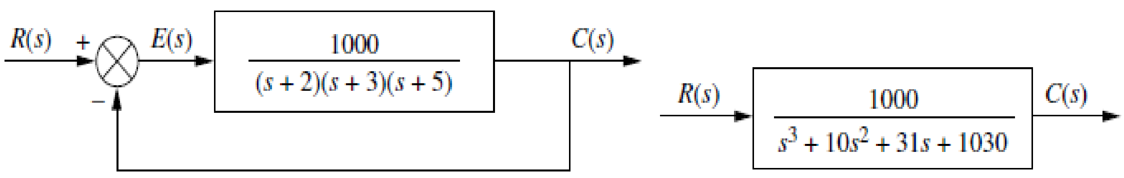
where all the coefficients are positive numbers. The array of coefficients becomes:

$$
\begin{array}{lcc}
s^3 & a_0 & a_2 \\
s^2 & a_1 & a_3 \\
s^1 & \dfrac{a_1 a_2 - a_0 a_3}{a_1} & \\
s^0 & a_3 &
\end{array}
$$

∴ The system will be stable if $a_1 a_2 > a_0 a_3$

Ex 2: Make the Routh table for the system shown in Figure (a).

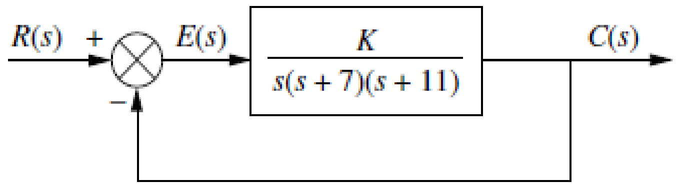
SOL: The first step is to find the equivalent closed-loop system because we want to test the denominator of this function, not the given forward transfer function, for pole location. Using the feedback formula, we obtain the equivalent system of Figure (b).



(a) (b)

$$
\begin{array}{llll}
s^3 & 1 & 31 & 0 \\
s^2 & \cancel{10}\;\; 1 & \cancel{1030}\;\; 103 & 0 \\
s^1 & \dfrac{-\begin{vmatrix} 1 & 31 \\ 1 & 103 \end{vmatrix}}{1} = -72 & \dfrac{-\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}}{1} = 0 & \dfrac{-\begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix}}{1} = 0 \\
s^0 & \dfrac{-\begin{vmatrix} 1 & 103 \\ -72 & 0 \end{vmatrix}}{-72} = 103 & \dfrac{-\begin{vmatrix} 1 & 0 \\ -72 & 0 \end{vmatrix}}{-72} = 0 & \dfrac{-\begin{vmatrix} 1 & 0 \\ -72 & 0 \end{vmatrix}}{-72} = 0
\end{array}
$$

Ex 3: Find the range of gain, K, for the system of Figure below that will cause the system to be stable, unstable, and marginally stable. Assume K > 0.



SOL: First find the closed-loop transfer function as:

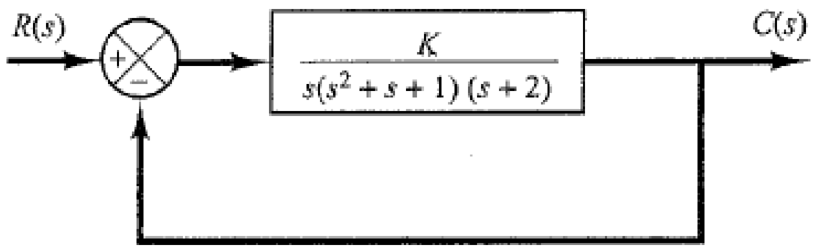
$$T(s) = \frac{K}{s^3 + 18s^2 + 77s + K}$$

Next form the Routh table shown as Table below

s^3	1	77
s^2	18	K
s^1	$\frac{1386 - K}{18}$	
s^0	K	

1. **Since K is assumed positive, we see that all elements in the first column are always positive except the s1 row. This entry can be positive, zero, or negative, depending upon the value of K. If K < 1386, all terms in the first column will be positive, then, the system be stable.**
2. **If K > 1386, the s1 term in the first column is negative then, the system be unstable.**
3. **If K =1386, we have an entire row of zeros. Then, the system is marginally stable**

Ex 4: determine the range of K for stability in Figure below.



Sol: The closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{K}{s(s^2 + s + 1)(s + 2) + K}$$

The characteristic equation is

$$s^4 + 3s^3 + 3s^2 + 2s + K = 0$$

The array of coefficients becomes

$$\begin{array}{llll} s^4 & 1 & 3 & K \\ s^3 & 3 & 2 & 0 \\ s^2 & \frac{7}{3} & K & \\ s^1 & 2 - \frac{9}{7}K & & \\ s^0 & K & & \end{array}$$

For stability, K must be positive, And all coefficients in the first column must be positive. Therefore,

$$\frac{14}{9} > K > 0$$

Root Locus Method-introduction

The relative stability and the transient performance of a closed loop system are directly related to the location of the closed-loop roots of the characteristic equation in the *s*-plane. It is frequently necessary to adjust one or more system parameters in order to obtain suitable root location.

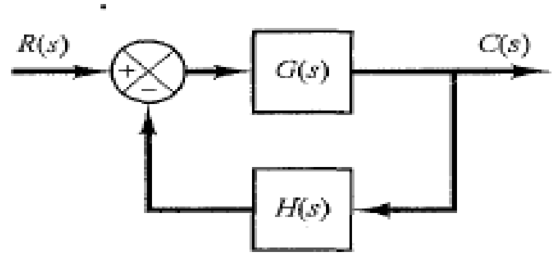
It is useful to determine the locus of roots in *s*-plane as a parameter varied since the roots is a function of the system's parameter. **The root locus technique** is a graphical method for sketching the locus of roots in the *s*-plane as a parameter is varied and has been utilized extensively in control engineering practice. It provides the engineer with a measure of the sensitivity of roots of the system a variation in parameter being considered. The root locus technique may be used to great advantage in conjunction with the Routh-Hurwitz criterion.

Root Locus Method Developed by Evans while he was a graduate student at UCLA. In designing a linear control system, we find that the root-locus method proves quite useful since it indicates the manner in which the open-loop poles and zeros should be modified so that the response meets system performance specifications. This method is particularly suited to obtaining approximate results very quickly.

General Rules for Constructing Root Loci

We shall now summarize the general rules and procedure for constructing the root loci of the system shown in Figure below. First, obtain the characteristic equation:

$$1 + G(s)H(s) = 0$$



1. Locate the poles and zeros o f G(s)H **(s)** on the s plane. The root-locus branches start from open-loop poles and terminate at zeros (finite zeros or zeros at infinity).
2. Determine No. of loci of the plot is equal to the order of C. E
3. Determine the root loci on the real **axis.** Root loci on the real axis are determined by open-loop poles and zeros lying on it. The complex-conjugate poles and zeros of the open-loop transfer function have no effect on the location of the root loci on the real axis because the angle contribution of a pair of complex-conjugate poles or zeros is 360" on the real axis.
4. Determine the asymptotes of root loci.

$$Angle\ of\ \ asymptotes = \psi = \frac{\pm 180^\circ (2k+1)}{n-m}$$

where

n-----> number of poles

m-----> number of zeros

For this Transfer Function $G(s)H(s) = \dfrac{K}{s(s+1)(s+2)}$

$$\psi = \frac{\pm 180^\circ (2k+1)}{3-0}$$

the number of distinct asymptotes is ***n – m***

5. Determine Point of intersection of asymptotes on real axis (or centroid of asymptotes) can be find as out

$$\boxed{\sigma = \frac{\sum poles - \sum zeros}{n-m}}$$

Ex: Determine the *asymptotes* of the root loci.

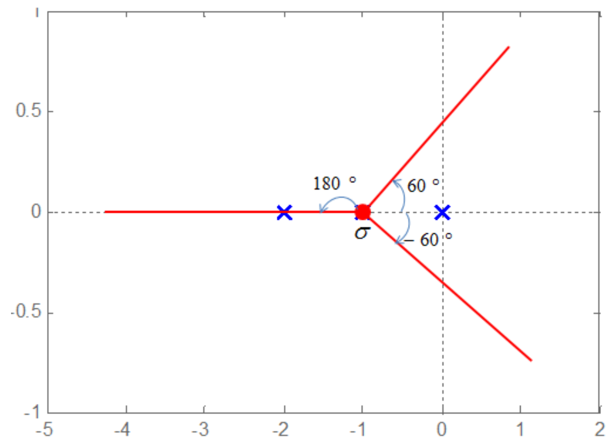
For

$$G(s)H(s)=\frac{K}{s(s+1)(s+2)}$$

$$\sigma=\frac{(0-1-2)-0}{3-0}$$

$$\sigma=\frac{-3}{3}=-1$$

$$\psi=60^\circ,-60^\circ,180^\circ$$



6. Find the breakaway and break-in points.

- **The breakaway point** corresponds to a point in the s plane where multiple roots of the characteristic equation occur. It is the point from which the root locus branches leaves real axis and enter in complex plane.
- **The break-in point** corresponds to a point in the s plane where multiple roots of the characteristic equation occur. It is the point where the root locus branches arrives at real axis.

- The breakaway or break-in points can be determined from the roots of

$$\frac{dK}{ds}=0$$

- It should be noted that not all the solutions of dK/ds=0 correspond to actual breakaway points.
- If a point at which dK/ds=0 is on a root locus, it is an actual breakaway or break-in point.
- Stated differently, if at a point at which dK/ds=0 the value of K takes a real positive value, then that point is an actual breakaway or break-in point.

Ex: Determine the *breakaway point* or *break-in point*.

$$G(s)H(s)=\frac{K}{s(s+1)(s+2)}$$

The characteristic equation of the system is

$$1+G(s)H(s)=1+\frac{K}{s(s+1)(s+2)}=0$$

$$\frac{K}{s(s+1)(s+2)}=-1$$

$$K=-[s(s+1)(s+2)]$$

The breakaway point can now be determined as

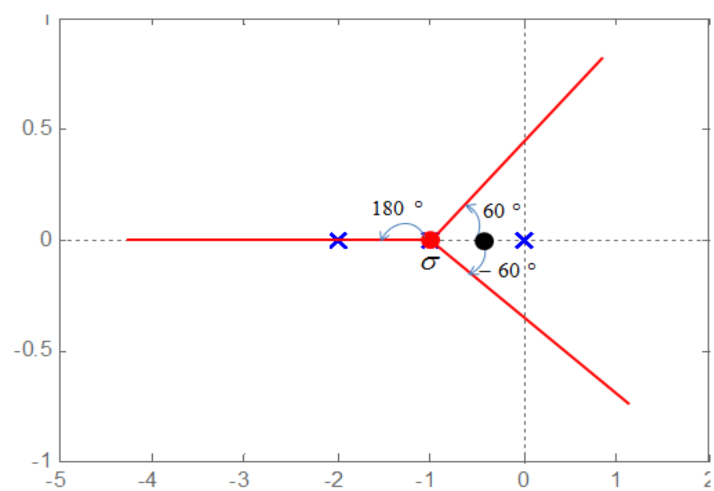
$$\frac{dK}{ds}=-\frac{d}{ds}[s(s+1)(s+2)]=0 \longrightarrow \begin{aligned} s&=-0.4226\\ &=-1.5774\end{aligned}$$

In fact, evaluation of the values of K corresponding to

s=–0.4226 and s=–1.5774 yields

$$K=0.3849, \qquad \text{for } s=-0.4226$$
$$K=-0.3849, \qquad \text{for } s=-1.5774$$

$$s=-0.4226$$



7. Determine the points where root loci cross the imaginary axis.

The points where the root loci intersect the *jw* axis can be found easily by:

(a) use of Routh's stability criterion

(b) letting s = *jw* in the characteristic equation, equating both the real part and the imaginary part to zero, and solving for *w* and K. The values of *w* thus found give the frequencies at which root loci cross the imaginary axis. The K value corresponding to each crossing frequency gives the gain at the crossing point.

EX: Determine the points where root loci cross the imaginary axis for C. E below:

$$s^3 + 3s^2 + 2s + K = 0$$

Sol: <u>first method:</u>

The Routh Array Becomes

$$\begin{array}{lcc} s^3 & 1 & 2 \\ s^2 & 3 & K \\ s^1 & \dfrac{6 - K}{3} & \\ s^0 & K & \end{array}$$

- **The value(s) of K that makes the system marginally stable is 6.**
- **The crossing points on the imaginary axis can then be found by solving the auxiliary equation obtained from the s^2 row, that is,**

$$3s^2 + K = 3s^2 + 6 = 0$$

Which yields

$$s = \pm j\sqrt{2}$$

<u>Second method</u>

An alternative approach is to let s=jω in the characteristic equation, equate both the real part and the imaginary part to zero, and then solve for ω and K.

- For present system the characteristic equation is

$$s^3+3s^2+2s+K=0$$

$$(j\omega)^3+3(j\omega)^2+2j\omega+K=0$$

$$(K-3\omega^2)+j(2\omega-\omega^3)=0$$

$$(K-3\omega^2)+j(2\omega-\omega^3)=0$$

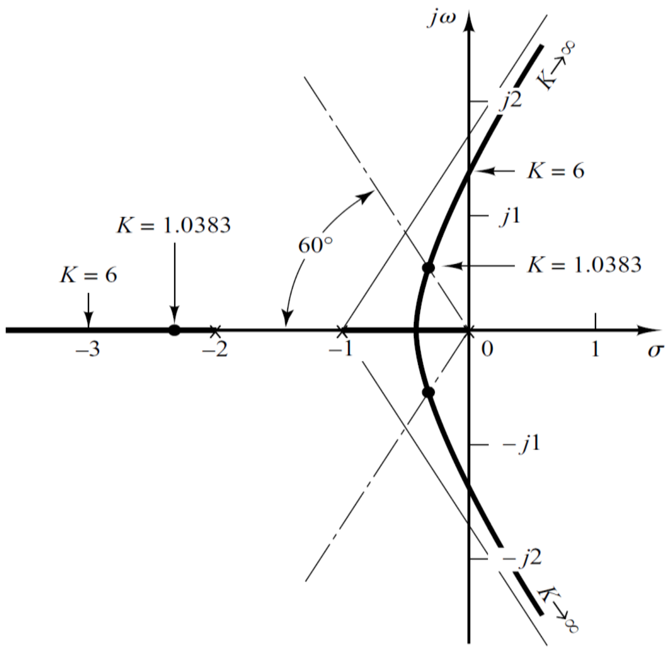
Equating both real and imaginary parts of this equation to zero

$$(2\omega-\omega^3)=0$$

$$(K-3\omega^2)=0$$

Which yields

$$\omega=\pm\sqrt{2},\qquad K=6\qquad \text{or}\qquad \omega=0,\qquad K=0$$



8. Determine the angle of departure (angle of arrival) of the root locus from a complex pole (at a complex zero). To sketch the root loci with reasonable accuracy, we must find the directions of the root loci near the complex poles and zeros. If a test point is chosen and moved in the very vicinity of a complex pole (or complex zero), the sum of the angular contributions from all other poles and zeros can be considered to remain the same. Therefore, the angle of departure (or angle of arrival) of the root locus from a complex pole (or at a complex zero) can be found by subtracting from 180" the sum of all the angles of vectors from all other poles and zeros to the complex pole (or complex zero) in question, with appropriate signs included.

Angle of departure from a complex pole=180 - (sum of the angles of vectors to a complex pole in question from other poles) + (sum of the angles of vectors to a complex pole in question from zeros)

Angle of arrival at a complex zero = 180 - (sum of the angles of vectors to a complex zero in question from other zeros) + (sum of the angles of vectors to a complex zero in question from poles)

Ex1: sketch the root locus of second order equation

$$G(s)H(s) = \frac{k(s+4)}{(s+1)(s+2)}$$

Sol:

1.We have two poles: p1=-1 , p2=-2 $\longrightarrow$ n=2

We have one zero z1=-4 $\longrightarrow$ m=1

2. the number of distinct asymptotes is ***n – m=2-1=1***

3. $$Angle\ of\ \ asymptotes = \psi = \frac{\pm 180°(2k+1)}{n-m}$$

= 180

4. $$\sigma = \frac{\sum poles - \sum zeros}{n-m} = \frac{-2-1}{1} = \frac{-3}{1} = -3$$

5. *breakaway point* or *break-in point*.

$1 + G(s)H(s) = 0$

$$1 + G(s)H(s) = 1 + \frac{K(s+4)}{(s+1)(s+2)} = 0$$

$$\left[(s+1)(s+2)\right] + k(s+4) = 0$$

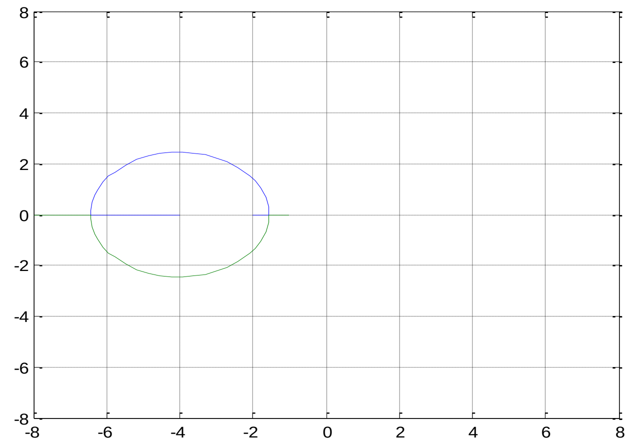
$$K = -\left[\frac{(s+1)(s+2)}{(s+4)}\right] = -\left[\frac{S^2 + 3S + 2}{S+4}\right]$$

$$\frac{dK}{ds} = \left[\frac{(S+4)(2S+3) - (S^2 + 3S + 2)}{(S+4)^2}\right] = 0$$

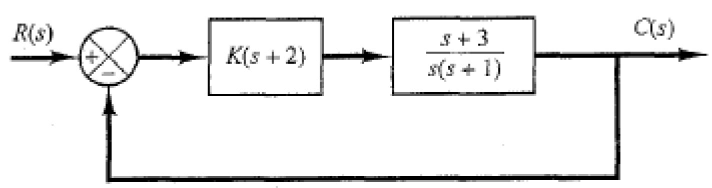
$$2s^2 + 11s + 12 - s^2 - 3s - 2 = 0$$

$$s^2 + 8s + 10 = 0$$

$$s = -1.55, or s = -6.45$$



Ex 2: Sketch the root loci of the control system shown in Figure below.



Solution:
The procedure for plotting the root loci is as follows:

1. Locate the open-loop poles and zeros on the complex plane. Root loci exist on the negative real axis between 0 and -1 and between -2 and -3.

2. The number of open-loop poles and that of finite zeros are the same. This means that there are no asymptotes in the complex region of the s plane.

3.Determine the breakaway and break-in points. The characteristic equation for the system is

$$1 + \frac{K(s + 2)(s + 3)}{s(s + 1)} = 0$$

or

$$K = -\frac{s(s + 1)}{(s + 2)(s + 3)}$$

The breakaway and break-in points are determined from

$$\frac{dK}{ds} = -\frac{(2s+1)(s+2)(s+3) - s(s+1)(2s+5)}{\left[(s+2)(s+3)\right]^2}$$

$$= -\frac{4(s+0.634)(s+2.366)}{\left[(s+2)(s+3)\right]^2}$$

$$= 0$$

as follows:

$$s = -0.634, \qquad s = -2.366$$

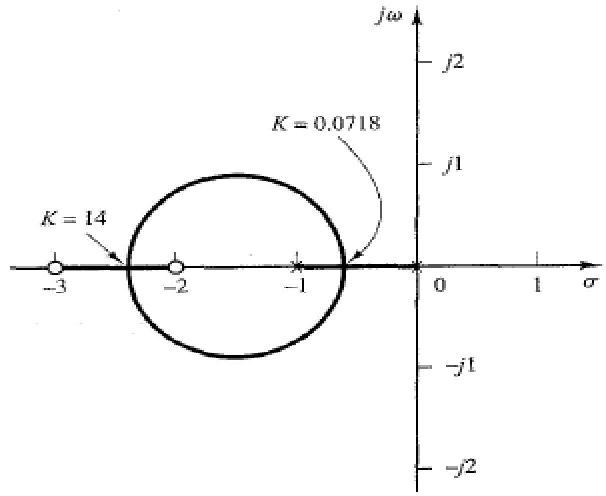
Notice that both points are on root loci. Therefore, they are actual breakaway or break-in points. At point s = -0.634, the value of K is

$$K = -\frac{(-0.634)(0.366)}{(1.366)(2.366)} = 0.0718$$

Similarly, at $s = -2.366$,

$$K = -\frac{(-2.366)(-1.366)}{(-0.366)(0.634)} = 14$$

Because point s = -0.634 lies between two poles, it is a breakaway point, and because point s = -2.366 lies between two zeros, it is a break-in point.)



Ex 3: Sketch the root loci of $G(s)H(s) = \frac{(s+2)}{(s+3)(s^2+2s+2)}$

1.We have two poles: p1=-3 , p2=-1+j, p3=-1-j $\longrightarrow$ n=3

We have one zero z1=-2 $\longrightarrow$ m=1

2. the number of distinct asymptotes is ***n – m=3-1=2***

3. $$Angle\ of\ asymptotes = \psi = \frac{\pm 180°(2k+1)}{n-m}$$
$$= \pm 90$$

4. $$\sigma = \frac{\sum poles - \sum zeros}{n-m} = \frac{-3-1+j-1-j+2}{3-1} = \frac{-3}{2} = -1.5$$

Frequency-Response Methods and Stability

The frequency response of a system is defined as the steady-state response of the system to a sinusoidal input signal. The sinusoid is a unique input signal, and the resulting output signal for a linear system, as well as signals throughout the system, is sinusoidal in the steady-state. Frequency-response methods were developed in 1930s and 1940s by Nyquist, Bode, Nichols, and many others. The frequency-response methods are most powerful in conventional control theory. They are also indispensable to robust control theory

Presenting Frequency-Response Characteristics in Graphical Forms:

The sinusoidal transfer function, a complex function of the frequency w, is characterized by its magnitude and phase angle, with frequency as the parameter. There are three commonly used representations of sinusoidal transfer functions:

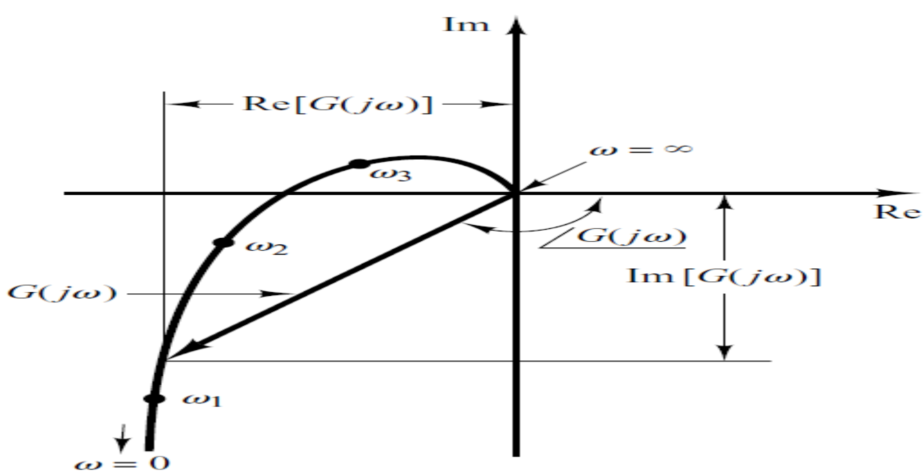
1. Bode diagram or logarithmic plot

2. Nyquist plot or polar plot

3. Log-magnitude-versus-phase plot (Nichols plots)

1. Polar Plots

The polar plot of a sinusoidal transfer function G(jω) is a plot of the magnitude of G(jω) versus the phase angle of G(jω) on polar coordinates as ω is varied from zero to infinity. Thus, the polar plot is the locus of vectors $|G(jw)|\angle G(jw)$ as ω is varied from zero to infinity.



Polar Plot

Each point on the polar plot of G(jω) represents the terminal point of a vector at a particular value of ω. The projections of G(jω) on the real and imaginary axes are its real and imaginary components.

- the frequency response can be calculated by replacing *s* in the transfer function by *jw*.
- It will also be shown that the steady-state response can be given by

  G(jω)=$R\llcorner\varphi$, where R is the amplitude ratio of the output and input sinusoids and φ is the phase shift between the input sinusoid and the output sinusoid
- Nyquist Stability Plot must be a closed contour

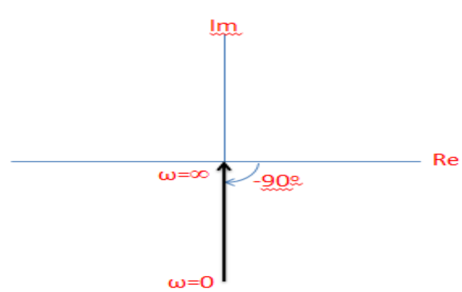
Procedure of Nyquist Plot

1. express the magnitude and phase equations in terms of w
2. Estimate the magnitude and phase for different values of w
3. Plot the curve and determine required performance metrics

Transfer Function Component Representation:

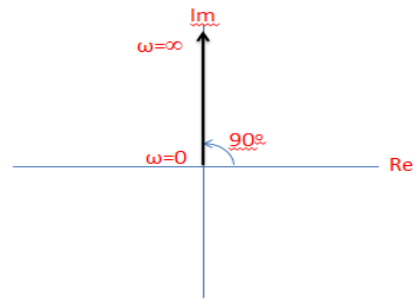
The polar plot of G(jω)=1/jω is the negative imaginary axis, since $G(j\omega)=\frac{1}{j\omega}$

In polar form $G(j\omega)=\frac{1}{\omega}\angle -90^\circ$



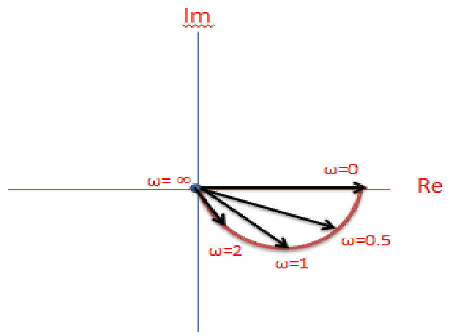
The polar plot of G(jω)=jω is the positive imaginary axis, since $G(j\omega)=j\omega$

In polar form $G(j\omega) = \omega \angle 90^\circ$



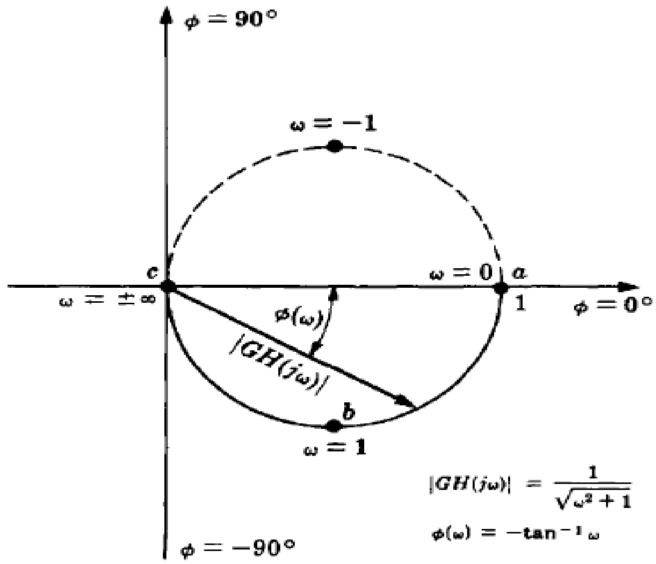
For the sinusoidal transfer function

$$G(j\omega) = \frac{1}{j\omega + 1}$$



ω	$\lvert G(j\omega) \rvert (R)$	$\angle G(j\omega)(\Phi)$
0	1	0^o
0.5	0.9	-26^o
1	0.7	-45^o
2	0.4	-63^o
∞	0	-90

The graph for above example is the mirror image about the diameter **of this** semicircle. **It is** shown in Fig below by a dashed line.



Example1: Draw the polar plot of following open loop transfer function.

$$G(s) = \frac{1}{s(s+1)}$$

SOL:

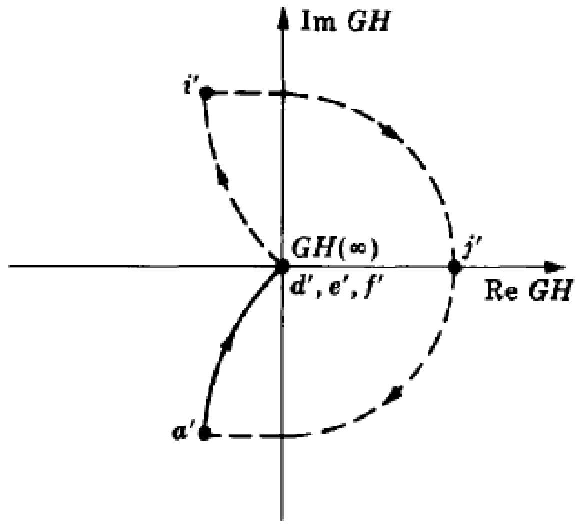
$$Put \;\; s = j\omega \longrightarrow G(j\omega) = \frac{1}{j\omega(j\omega+1)}$$

$$G(j\omega) = \frac{1}{-\omega^2 + j\omega} \longrightarrow G(j\omega) = \frac{1}{-\omega^2 + j\omega} \times \frac{-\omega^2 - j\omega}{-\omega^2 - j\omega}$$

$$G(j\omega) = \frac{-\omega^2 - j\omega}{\omega^4 + \omega^2} \longrightarrow G(j\omega) = \frac{-\omega^2}{\omega^4 + \omega^2} - j\frac{\omega}{\omega^4 + \omega^2}$$

$$G(j\omega) = \frac{-1}{\omega^2 + 1} - j\frac{1}{\omega(\omega^2 + 1)}$$

ω	Re	Im
0	∞	∞
0.1	-1	-10
0.5	-0.8	-1.6
1	-0.5	-0.5
2	-0.2	-0.1
3	-0.1	-0.03
∞	0	0



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